

Superluminal neutrinos from Lorentz-violating dimension-5 operators

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We consider Lorentz- and CPT-violating dimension-5 operators to address the issue of superluminal neutrinos recently pointed out in OPERA experiments. We assume these operators in the photon and neutrino sectors coupled to Lorentz-violating backgrounds in a preferred frame defined by a time-like direction. We show that such operators can produce a curve with OPERA's slope that fits OPERA, MINOS and supernova SN1987a data.

In this letter we consider Extended Myers-Pospelov dimension-5 operators in order to consider physics in a preferred frame with time-like direction in the presence of a Lorentz-violating background n_μ to address the issue of the superluminal neutrinos detected in the OPERA's experiments very recently [1] — for some recent theoretical developments see, for example [2–5]. We derive the dispersion relations associated to the effective Lagrangian for Dirac and Maxwell terms supplemented by dimension-5 operators given in the form

$$\mathcal{L}_{eff} = \bar{\psi}(i\hat{\not{D}} - m)\psi + \tilde{g}\bar{\psi}\not{\gamma}_5\hat{D}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + g\epsilon^{\alpha\mu\lambda\rho}n_\lambda\hat{D}F_{\rho\alpha}A_\mu + \dots \quad (1)$$

where (\dots) means interacting terms, $\tilde{g} = \eta/M$ and $g = \xi/M$ with η, ξ dimensionless parameters. The dimension-5 operator in the electromagnetic sector is CPT - odd and even under charge conjugation, whereas the dimension-5 operator in the fermionic sector breaks CPT and is even under charge conjugation, M is the mass where new physics such as Lorentz and CPT symmetry violation emerges and \hat{D} is a derivative operator given by

$$\hat{D} = (\partial \cdot n)^2 - \partial^2 n^2. \quad (2)$$

This is the Myers-Pospelov operator [6] suitably extended in the present study to affect the superluminality behavior at large momenta. Note this reduces to the original operator for light-like backgrounds, i.e., for $n^2 = 0$. The fermionic sector with $\tilde{g}\hat{D}\not{\gamma}_5 \rightarrow -\not{\gamma}_5$ reduces to CPT - odd extended QED by Colladay and Kostelecky [7], which has been recently considered in superluminal neutrino issues [8]. Similarly, the electromagnetic sector with $gn_\mu\hat{D} \rightarrow -\kappa_\mu$ reduces to the Maxwell and Carroll-Field-Jackiw model [9]. However, it was shown in [10, 11] that this theory in time-like background does not produce superluminal velocities. So at this level one cannot expect to fit OPERA's data [1]. Thus, we should go to higher dimensional operators as in (1). Although such operators can produce superluminal velocities they comprise a Lorentz invariant Lagrangian except by the appearance of the Lorentz-violating background four vector n^μ [6, 7], which can also be understood as a vacuum expectation value (VEV) of a vector field [3, 4]. For the sake of simplicity, we choose time-like backgrounds, which is not necessarily the frame of the neutrinos.

Let us now derive the dispersion relation associated to extended Myers-Pospelov dimension-5 operator in the fermionic sector. The equation of motion takes the form

$$(i\hat{\not{D}} - m - \tilde{g}\hat{D}\not{\gamma}_5)\psi = 0. \quad (3)$$

After a straightforward algebra we find that the free continuous spectrum is governed by the following dispersion relation

$$(k^2 - m^2 - \tilde{g}^2 n^2 P_n^2)^2 - 4\tilde{g}^2 P_n^3 = 0, \quad (4)$$

where

$$P_n = (n \cdot k)^2 - n^2 k^2. \quad (5)$$

For related issues in the original Myers-Pospelov model see, e.g., [12–14] — see also [15]. Hence, settling a time-like direction in the Lorentz symmetry breaking background $n_\mu = (n_0, 0, 0, 0)$, we have

$$(\omega^2 - |\vec{k}|^2 - m^2 - \tilde{g}^2 n_0^6 |\vec{k}|^4)^2 - 4\tilde{g}^2 n_0^6 |\vec{k}|^6 = 0, \quad (6)$$

whose solutions are

$$\omega_\pm = \pm \sqrt{|\vec{k}|^2 (1 \pm \tilde{g} n_0^3 |\vec{k}|)^2 + m^2}. \quad (7)$$

We assume here the neutrino velocity is given by the group velocity determined from the fermionic dispersion relation (7), that is

$$v_\nu = \frac{\partial \omega_+}{\partial |\vec{k}|} = \frac{|\vec{k}|(1 \pm \tilde{g}n_0^3|\vec{k}|)(1 \pm 2\tilde{g}n_0^3|\vec{k}|)}{\sqrt{|\vec{k}|^2(1 \pm \tilde{g}n_0^3|\vec{k}|)^2 + m_\nu^2}}. \quad (8)$$

Now expanding in large momenta $|\vec{k}|^2 \gg m_\nu^2$, but keeping $\tilde{g}n_0^3|\vec{k}| \ll 1$ we find

$$v_\nu \simeq 1 \pm 2\tilde{g}n_0^3|\vec{k}| - \frac{m_\nu^2}{2|\vec{k}|^2}(1 - 4\tilde{g}^2n_0^6|\vec{k}|^2). \quad (9)$$

If we maintain only linear terms in \tilde{g} we find (for the “plus sign” sector)

$$v_\nu - 1 \sim 2\tilde{g}n_0^3|\vec{k}| - \frac{m_\nu^2}{2|\vec{k}|^2}. \quad (10)$$

Note that for massless or almost massless fermions as in the case of neutrinos we have possibility of superluminal velocities. We can indeed estimate the Lorentz violating coupling \tilde{g} if we know the energy $E_\nu \sim |\vec{k}|$ and the mass m_ν of the superluminal neutrino satisfying the bound

$$4\tilde{g}n_0^3|\vec{k}|^3 > m_\nu^2. \quad (11)$$

As we shall see below, this is easily satisfied by OPERA’s data [1] where $\tilde{\alpha} \equiv \tilde{g}n_0^3 \sim (1.7 \times 10^6 \text{ GeV})^{-1}$.

Let us now write the deviation of the neutrino velocity from the light speed (in the vacuum) as a function of momenta according to the curve with slope $\tilde{\alpha}$

$$\frac{v_\nu - c}{c} \simeq 2\tilde{\alpha}|\vec{k}|, \quad (12)$$

where we are choosing $c = 1$. For OPERA experiments $v_\nu - 1 \sim 10^{-5}$ with $E_\nu \sim |\vec{k}| \sim 17 \text{ GeV}$ we find the slope

$$\tilde{\alpha} \sim \frac{1}{1.7 \times 10^6 \text{ GeV}} \sim 10^{-22} \text{ m}, \quad (13)$$

which sets our Lorentz symmetry breaking parameter $\tilde{\alpha}$ to test superluminality up to the energy 1000 TeV, the energy scale present in primary cosmic rays originated in the explosion of massive stars [16]. Now substituting (13) into (12) we get to the curve

$$\frac{v_\nu - c}{c} \sim 10^{-5} \left(\frac{|\vec{k}|}{17 \text{ GeV}} \right). \quad (14)$$

Note this also agrees with MINOS experiment [17] with energy $E_\nu \sim |\vec{k}| \sim 3 \text{ GeV}$. To compare OPERA with supernova SN1987a measurements [18] we use $E_\nu \sim |\vec{k}| \sim 10 \text{ MeV}$ to obtain

$$\frac{v_\nu - c}{c} \sim 10^{-5} \left(\frac{10^{-3}}{1.7} \right), \quad (15)$$

that is consistent with the SN1987a bound $|v - c|/c < 2 \times 10^{-9}$ [18]. We conclude that the above constraints on the deviation from the light speed in superluminal neutrinos obey a *linear curve fitting* as a function of the energy with OPERA’s slope $\tilde{\alpha} \sim 0.5 \times 10^{-6} \text{ GeV}^{-1}$. This has also been noticed in other recent investigations [19–21].

Similarly we can find the dispersion relation for the electromagnetic sector through its corresponding equation of motion

$$(\partial^2 \eta^{\mu\nu} - \partial^\mu \partial^\nu - 2g\hat{D}\varepsilon^{\nu\mu\lambda\rho}n_\lambda \partial_\rho)A_\nu = 0. \quad (16)$$

The photon dispersion relation in the time-like Lorentz-violating background is then given by

$$(\omega^2 - |\vec{k}|^2)^2 - 4g^2n_0^6|\vec{k}|^6 = 0. \quad (17)$$

Solving this equation we obtain the following solutions

$$\omega_{\pm}(\vec{k}) = \pm |\vec{k}| \sqrt{1 \pm 2gn_0^3|\vec{k}|}. \quad (18)$$

The group velocity here determines the photon velocity

$$v_{\gamma} = \frac{\partial \omega_{+}}{\partial |\vec{k}|} = \frac{1 \pm 3gn_0^3|\vec{k}|}{\sqrt{1 \pm 2gn_0^3|\vec{k}|}}. \quad (19)$$

For large momenta but keeping $|2gn_0^3\vec{k}| \ll 1$ we find (for the “plus sign” sector)

$$v_{\gamma} - 1 \simeq 2gn_0^3|\vec{k}|. \quad (20)$$

Let us now use the group velocity for the photon given in (20) to write the deviation of the neutrino velocity from the light speed (in the presence of the Lorentz-violating background) as a function of momenta (up to linear terms)

$$\frac{v_{\nu} - v_{\gamma}}{v_{\gamma}} \simeq 2(\tilde{\alpha} - \alpha)|\vec{k}|. \quad (21)$$

Recall that $\tilde{\alpha} \equiv \tilde{g}n_0^3$, $\alpha \equiv gn_0^3$, $\tilde{g} = \eta/M$ and $g = \xi/M$, where we shall assume $n_0 = 1$ from now on. The expected superluminality of the neutrino (with mass $m_{\nu} \sim 0.2$ eV and energy $E_{\nu} \sim |\vec{k}| \sim 17$ GeV) is negligible

$$\frac{v_{\nu} - v_{\gamma}}{v_{\gamma}} \simeq 10^{-21}. \quad (22)$$

By using (21) allows us to set a bound to the couplings η and ξ that measure the strength of the time-like Lorentz-violating background acting into the neutrino and photon sectors. Thus, at the Planck scale $M \sim 10^{19}$ GeV it follows that

$$\eta - \xi \simeq \frac{10^{-21}M}{2 \times 17 \text{ GeV}} \sim 2.5 \times 10^{-4}. \quad (23)$$

The bound for η can be found through the bound $\xi \sim 10^{-6}$ for photons as follows

$$\frac{\eta}{\xi} \simeq 1 + \frac{2.5 \times 10^{-4}}{\xi} \sim 10^2, \quad (24)$$

which corresponds to $\eta \sim 10^{-4}$ for neutrinos. This means that at the Planck scale the neutrino interacts with the Lorentz violating background about 100 times as much as the photon interacts. Up to one order of magnitude these bounds approach those derived from renormalization group equations [6] — see also [22] for other bounds via dimension-4 operators in purely space-like backgrounds.

If the couplings η and ξ change with energy keeping the difference $\eta - \xi$ then we consider it as the slope of the curve

$$\delta v_{\nu} = \frac{v_{\nu} - v_{\gamma}}{v_{\gamma}} \simeq 2(\eta - \xi) \frac{|\vec{k}|}{M} \sim 5 \times 10^{-4} \frac{|\vec{k}|}{M}. \quad (25)$$

It is interesting to note that if we bring the Planck scale to the TeV scale, i.e., $M = M_{\text{Earth}} \sim 1$ TeV and $|\vec{k}| \sim 17$ GeV one finds the OPERA’s result, i.e., about $\delta v_{\nu} \sim 10^{-5}$, for the deviation of the neutrino velocity from the light speed. Furthermore, for the scale $M = M_{\text{Astro}} > 1000$ TeV it is sufficient to avoid astrophysical constraints since one can readily find $\delta v_{\nu} < 10^{-9}$. This is in accord with [4] since in the latter case the strength of the Lorentz-violating background $\tilde{\alpha}_{\text{Astro}} \sim 1/M_{\text{Astro}}$ is about 10^4 times smaller than $\tilde{\alpha}_{\text{Earth}} \sim 1/M_{\text{Earth}}$ in the former case. One should note that the formula (25) can also be readily applied to *electrons*. Finally, noticed that for photons one makes use of equation (20) and $g \sim \tilde{g}$ ($\xi \sim \eta$) on the Earth — this is consistent with (23) — to find that $\delta v_{\gamma} \sim 10^{-5}$ at $|\vec{k}| \sim 17$ GeV, a result as also found in [4].

To bring the Planck scale to lower scales one should consider extra-dimensions. In the five-dimensional Randall-Sundrum scenario [23] one finds $M_5 \approx M_{\text{Planck}}$ for the warp factor e^{kL} about 10^{15} and $M \equiv M_5 e^{-kL} \sim 1$ TeV on the TeV (i.e., IR or Standard Model) brane.

Our Lagrangian is Lorentz invariant except by the appearance of the Lorentz-violating background four vector n^{μ} . This is in the same class of the theories [3, 4, 6, 7]. As such, we can always choose a rest frame for the neutrino to study its decay to other particles. Because of the Lorentz invariance the energy-momentum is locally conserved and decay of highly energetic neutrinos into neutrinos and other particles with lower energies is forbidden [3]. This should be enough to evade the Cohen-Glashow bound [5]. Furthermore, we can fine tune ξ and η in order to make the Lorentz-violating background much stronger on the Earth than interstellar scale to avoid all the astrophysical constraints on Lorentz violation and Cohen-Glashow bound as well discussed in [4].

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